TRINITY COLLEGE

Semester One Examination, 2018

Question/Answer booklet

MATHEMATICS APPLICATIONS UNIT 3, 4

Section Two: Calculator-assumed

SOLUTIONS

Student number: In figures | | | | | | | | |

In words _____

Your name _____

Time allowed for this section

Reading time before commencing work: ten minutes
Working time: one hundred minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	10	10	100	98	65
				Total	100

Instructions to candidates

1. The rules for the conduct of Trinity College examinations are detailed in the *Instructions to Candidates* distributed to students prior to the examinations. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet.

3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

4. Supplementary pages for the use of planning/continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

5. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you do not use pencil, except in diagrams.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

See next page

Section Two: Calculator-assumed **65% (98 Marks)**

This section has **ten (10)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 9 **(7 marks)**

(a) The first three terms, in order, of a geometric sequence are 900, 540 and 324.

(i) Deduce a rule for the n^{th} term of this sequence. (2 marks)

Solution
$r = 540 \div 900 = 0.6$ $T_n = 900(0.6)^{n-1}$
Specific behaviours
✓ calculates ratio ✓ writes rule

(ii) Calculate the 6^{th} term of the sequence. (1 mark)

Solution
$T_6 = 69.984$
Specific behaviours
✓ exact value of term

(b) The first three terms, in order, of an arithmetic sequence are 1.5, 4.2 and 6.9.

(i) The rule for the n^{th} term of this sequence is $T_n = an + b$. Determine the values of a and b. (3 marks)

Solution
$d = 4.2 - 1.5 = 2.7$ $T_n = 1.5 + (n-1)(2.7)$ $= 2.7n - 1.2$ $a = 2.7, \quad b = -1.2$
Specific behaviours
✓ calculates difference ✓ writes rule ✓ simplifies and states values

(ii) Calculate the 316^{th} term of the sequence. (1 mark)

Solution
$T_{316} = 852$
Specific behaviours
✓ exact value of term

See next page

Question 10 **(10 marks)**

A public relations company was tasked with determining whether a person's support for a sugary drinks tax could be associated with their interest in the news.

The company carried out a telephone survey, where people could respond to two questions as shown in the following table:

Question	Choice of response
Are you interested in the news?	Yes or No
Do you support a sugary drinks tax?	Yes, No or Undecided

The responses to the telephone survey are summarised in this table:

		Support for a sugary drinks tax		
		Yes	No	Undecided
Interest in news	Yes	556	367	233
	No	196	136	146

(a) Calculate the number of people who

(i) answered yes to being interested in the news. (1 mark)

Solution
$556 + 367 + 233 = 1\,156$
Specific behaviours
✓ number

(ii) responded to the survey. (1 mark)

Solution
$1156 + 478 = 1\,634$
Specific behaviours
✓ number

(b) If there was no association between interest in the news and support for a sugary drinks tax, should the company expect a larger percentage of those who are interested in the news to support a sugary drinks tax compared to those who are not interested in the news? Explain your answer. (2 marks)

Solution
No. If there was no association then the percentages would be the same.
Specific behaviours
✓ indicates no ✓ explanation indicating same percentages

See next page

(c) Complete the two-way table below to show the associated **row** percentages for the previous table, rounding percentages to the nearest whole number. (3 marks)

		Support for a sugary drinks tax		
		Yes	No	Undecided
Interest in news	Yes	48%	32%	20%
	No	41%	28%	31%

Solution
See table
Specific behaviours
✓ first row, ✓ second row, ✓ rounds correctly

(d) What percentage of those who are not interested in the news do not support a sugary drinks tax? (1 mark)

Solution
28%
Specific behaviours
✓ percentage from table

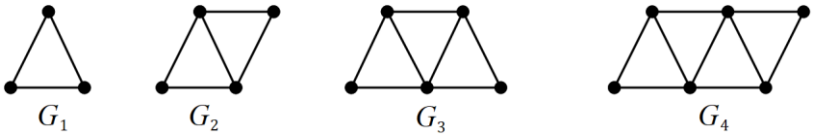
(e) In the context of the task they were given, how should the public relations company interpret the responses to their survey? (2 marks)

Solution
The PR company should conclude that an association exists between interest in the news and support for a sugary drinks tax. For example, 48% of those interested in the news supported a sugary drinks tax compared to only 41% of those who were not interested in the news.
Specific behaviours
✓ identifies an association exists ✓ justifies with reference to different percentages in at least one column

See next page

Question 11 **(8 marks)**

A sequence of four connected graphs is shown below.



(a) Complete the missing entries in the table below, where the vertex sum is the sum of the degrees of all the vertices in a graph. (2 marks)

Graph (n)	1	2	3	4
Vertices (V)	3	4	5	6
Vertex sum (S)	6	10	14	18

Solution
See table
Specific behaviours
✓ correct vertices, ✓ correct vertex sums

Assume the sequence of graphs continues indefinitely.

(b) A graph in the sequence has 11 faces. How many vertices does it have? (2 marks)

Solution
$11 - 1 = 10 \Rightarrow G_{10}$ $V_{10} = 3 + 9 \times 1 = 12$
Specific behaviours
✓ indicates graph number ✓ number of vertices

(c) Deduce the n^{th} term rule for S_n, the vertex sum of graph n. (2 marks)

Solution
$S_n = 6 + (n-1)(4)$ $= 4n + 2$
Specific behaviours
✓ indicates first term and difference ✓ correct rule

(d) A graph in the sequence has a vertex sum of 350. Determine the number of vertices this graph has. (2 marks)

Solution
$4n + 2 = 350 \Rightarrow n = 87$ $3 + 86 \times 1 = 89$ vertices
Specific behaviours
✓ uses rule from (c) to find n ✓ number of vertices

See next page

Question 12 **(8 marks)**

Data from the Australian Bureau of Statistics showing the quarterly beer production (in megalitres) over 23 quarters (December 2004 to June 2010) reveal a seasonal cycle with a period of 4 quarters. The 23 quarters are used to calculate 4-point centred moving averages.

(a) What is the purpose of calculating moving averages for such data. (2 marks)

Solution
To smooth the data and remove variation from cyclical factors, in order to identify a trend.
Specific behaviours
✓ mentions smoothing data ✓ mentions trend

(b) Using D (the deseasonalised data) and t (where $t=1$ for December 2004, $t=2$ for March 2005, etc.), the equation of the regression line that models the trend component is

$$D = 424.24 - 0.0077t$$

Describe what the slope of this line indicates about beer production. (2 marks)

Solution
Each quarter of beer production decreases by 0.0077 megalitres.
Specific behaviours
✓ mentions decreases ✓ uses the numerical value of 0.0077

(c) The seasonal proportions for the June quarters are

$$0.96, 0.90, 0.90, 0.92, 0.93$$

(i) What do these seasonal proportions indicate about beer production in the June quarter? (1 mark)

Solution
We expect lower beer production in June
Specific behaviours
✓ correct statement or reason

(ii) Calculate the seasonal index for June. (1 mark)

Solution
$\frac{0.96 + 0.90 + 0.90 + 0.92 + 0.93}{5} = 0.922$
Specific behaviours
✓ correct value

(d) Using an appropriate model, predict the number of megalitres of beer produced for the June quarter of 2013. Comment on your prediction. (3 marks)

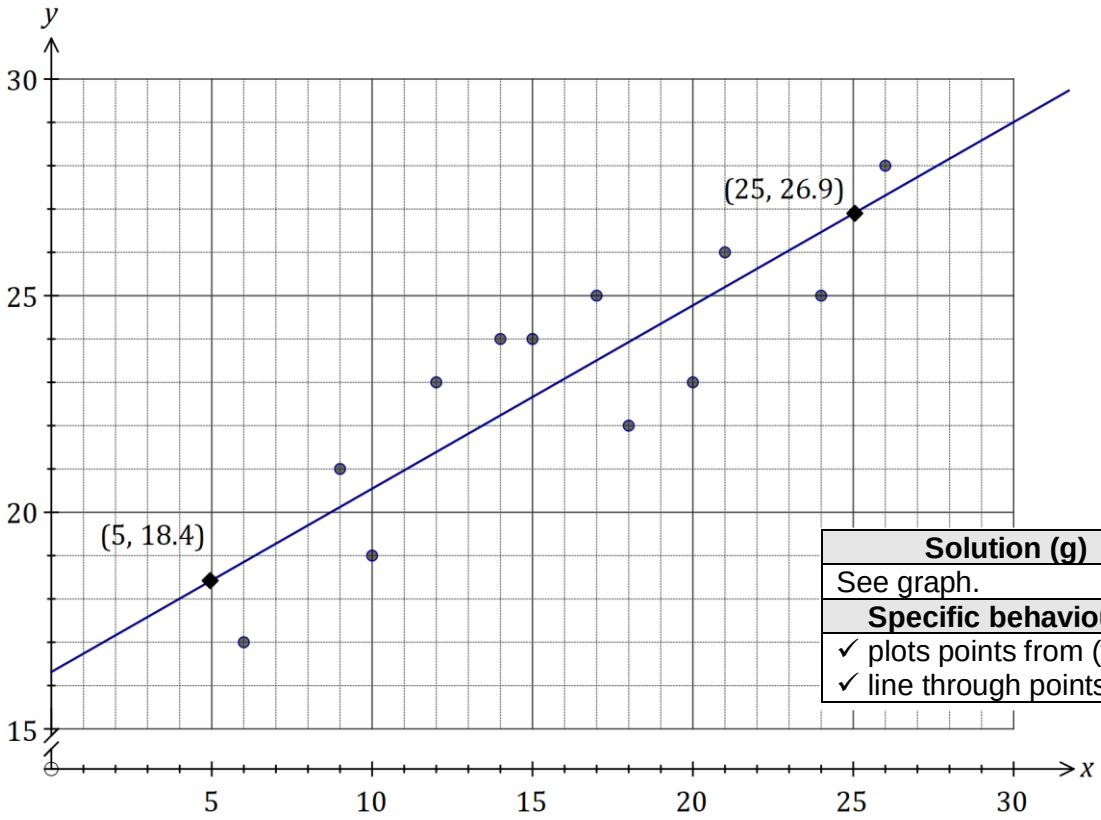
Solution
June 2013 gives $t = 35$ Prediction $= (424.24 - 0.0077 \times 35) \times 0.922 = 390.9\ ML$ This prediction is not reliable because it is 3 cycles away.
Specific behaviours
✓ correct *t* value ✓ correct prediction ✓ correct comment

See next page

Question 13 **(15 marks)**

A tomato grower added varying amounts of a liquid fertiliser (x ml) to the irrigation systems of twelve greenhouses and observed the resulting yield of tomatoes per plant (y kg). A sample of the data recorded is shown in the table and scatterplot below.

x (mL)	6	9	10	12	14	15	17	18	20	21	24	26
y (kg)	17	21	19	23	24	24	25	22	23	26	25	28



Solution (g)
See graph.
Specific behaviours
✓ plots points from (f)
✓ line through points

(a) Name the response variable. (1 mark)

Solution
y (yield per plant)
Specific behaviours
✓ correct variable

(b) For this data, calculate

(i) the correlation coefficient. (1 mark)

Solution
$r = 0.859$
Specific behaviours
✓ value that rounds to 0.86

(ii) the values of a and b in the equation of the least-squares line $y = ax + b$. (2 marks)

Solution
$a = 0.423, \quad b = 16.31$
Specific behaviours
✓ gradient (a)
✓ y-intercept (b)

See next page

(c) What percentage of the variation in the yield per plant can be explained by the variation in the amount of liquid fertiliser added? (1 mark)

Solution
$r^2 = 0.738$ which is 73.8%.
Specific behaviours
✓ states correct percentage

(d) If the amount of liquid fertiliser added to the irrigation system in a greenhouse was increased by one millilitre, what increase in the yield of tomatoes per plant can be expected? Explain your answer. (2 marks)

Solution
An increase of 0.423 kg per plant. This value if the gradient of the least-squares line.
Specific behaviours
✓ value ✓ indicates use of gradient

(e) If no liquid fertiliser was used, what yield of tomatoes per plant does the linear model predict? (1 mark)

Solution
A yield of 16.3 kg.
Specific behaviours
✓ uses value of y-intercept

(f) Use the equation of the least-squares line to calculate the value of y when $x = 5$ and when $x = 25$. (2 marks)

Solution
$x = 5, \quad y = 18.4$ $x = 25, \quad y = 26.9$
Specific behaviours
✓ one correct value ✓ both correct values

(g) Use your answers to part (f) to draw the least-squares line on the scatterplot. (2 marks)

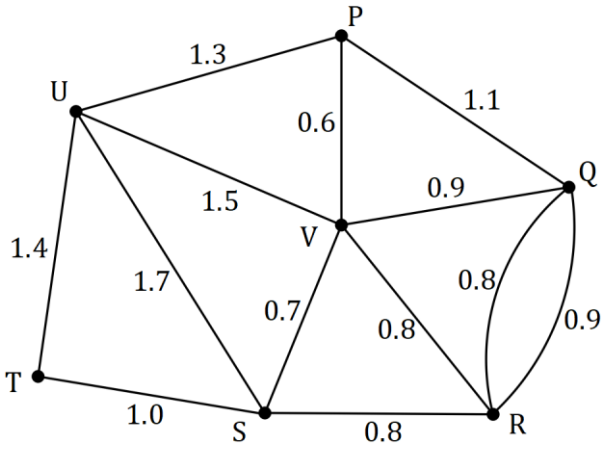
(h) Estimate the yield of tomatoes per plant when 8 ml of liquid fertiliser is added to the irrigation system and comment on the reliability of this value. (3 marks)

Solution
$x = 8$ ml, $\quad y = 19.7$ kg per plant Estimate is reliable as correlation is strong and it involves interpolation.
Specific behaviours
✓ any value that rounds to 20 kg ✓ considers correlation ✓ considers interpolation

See next page

Question 14 **(10 marks)**

The roads in a suburb are represented in the graph below, where the number on each edge is the length, in km, of the road.



(a) The network contains a semi-Eulerian trail that ends at P.

(i) Explain what semi-Eulerian means. (2 marks)

Solution
Trail along all edges just once, ending at different vertex to start.
Specific behaviours
✓ all edges once, ✓ different start and finish vertices

(ii) At which vertex must the trail start? (1 mark)

Solution
Vertex V
Specific behaviours
✓ correct vertex

(iii) Determine the number of edges in the trail. (1 mark)

Solution
13 edges
Specific behaviours
✓ correct number

(iv) How many times does the trail pass through vertex Q? (1 mark)

Solution
2 times
Specific behaviours
✓ correct number

See next page

(b) A worker needs to leave V, travel along each road once to inspect its surface and then return to V. Determine the minimum distance the worker must travel. (3 marks)

Solution
Sum of weights: 13.5 km Will end trail at P and then return 0.6 km to V Minimum distance: $13.5 + 0.6 = 14.1$ km
Specific behaviours
✓ indicates sum of all weights ✓ indicates distance to return to G ✓ correct minimum distance

(c) Another worker needs to walk along each road twice, once on each side of the road, as they deliver advertising pamphlets to houses. Determine, with justification, the minimum distance this worker must travel if they start and finish at S and the width of the roads is ignored. (2 marks)

Solution
Each edge is duplicated, so all vertices even and graph is now Eulerian - can start at any vertex and return after visiting all edges. Minimum distance is double sum of weights: $2 \times 13.5 = 27.0$ km
Specific behaviours
✓ reasoning ✓ correct distance

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Question 15 **(13 marks)**

Measurements of temperature for Perth were recorded over a period of three years. Each measurement is the mean maximum temperature over a three-month season.

Season	Time period (t)	Temperature (T) ($^\circ C$)	Moving average	Cycle mean	Seasonal proportion	Deseasonalise data (D)
Summer 2015	1	31.4			C	25.4
Autumn 2015	2	25.4		25.59	0.99	24.8
Winter 2015	3	19.7	25.6		0.77	25.7
Spring 2015	4	25.9	25.5		1.01	26.6
Summer 2016	5	31.2	25.3		1.29	25.2
Autumn 2016	6	25.1	24.7	B	1.04	24.6
Winter 2016	7	18.0	24.1		0.74	23.5
Spring 2016	8	22.7	24.0		0.93	23.2
Summer 2017	9	29.8	24.3		1.20	24.1
Autumn 2017	10	25.9	24.7		1.04	25.3
Winter 2017	11	19.5	24.9	24.89	0.78	25.5
Spring 2017	12	24.4			0.98	25.0
Summer 2018	13	A				24.0

(a) Using a 4-point centred moving average, calculate values for A, B and C in the table above. (3 marks)

Solution

$$\frac{0.5 \times 29.8 + 25.9 + 19.5 + 24.4 + 0.5 \times A}{4} = 24.9 \qquad A = 29.8$$

$$\frac{31.2 + 25.1 + 18.0 + 22.7}{4} = B \qquad B = 24.25$$

$$\frac{31.4}{25.59} = C \qquad C = 1.23$$

Specific behaviours

✓ correct value for *A*
✓ correct value for *B*
✓ correct value for *C*

See next page

(b) In 2014, one season had a Seasonal proportion of 0.96. Which season do you think it was? Explain your answer. (2 marks)

Solution
Spring because all seasonal proportions for Spring are between 0.93 and 1.01
Specific behaviours
✓ correct season (Spring) ✓ correct explanation

(c) The trend for this data can be modelled by the regression line

$$D = 25.45 - 0.09t$$

A newspaper article has stated that this provides evidence against global warming.

(i) What does the slope of the regression line indicate about the change in temperature over time? (2 marks)

Solution
Temperature is decreasing by $0.09°C$ per season
Specific behaviours
✓ mentions decreasing ✓ uses value of 0.09

(ii) Comment on the validity of the newspaper article's claim. (2 marks)

Solution
Not valid since measurements at other times may well contradict this short trend
Specific behaviours
✓ Not valid ✓ correct comment

(iii) A predicted mean temperature for Winter is 17.8°. Determine which year this prediction is from. (3 marks)

Solution
$Season\ index = \frac{0.77 + 0.74 + 0.78}{3} = 0.76$ $Pred = trend \times seasonal\ index$ $17.8 = (25.45 - 0.09t) \times 0.76$ $t = 22.54 \approx 23$ Therefore, during Winter 2020
Specific behaviours
✓ calculates seasonal index ✓ uses prediction to calculate value for t ✓ correct year

See next page

Question 16 **(9 marks)**

The temperature, T ℃, of an industrial oven n minutes after it is turned on can be modelled by

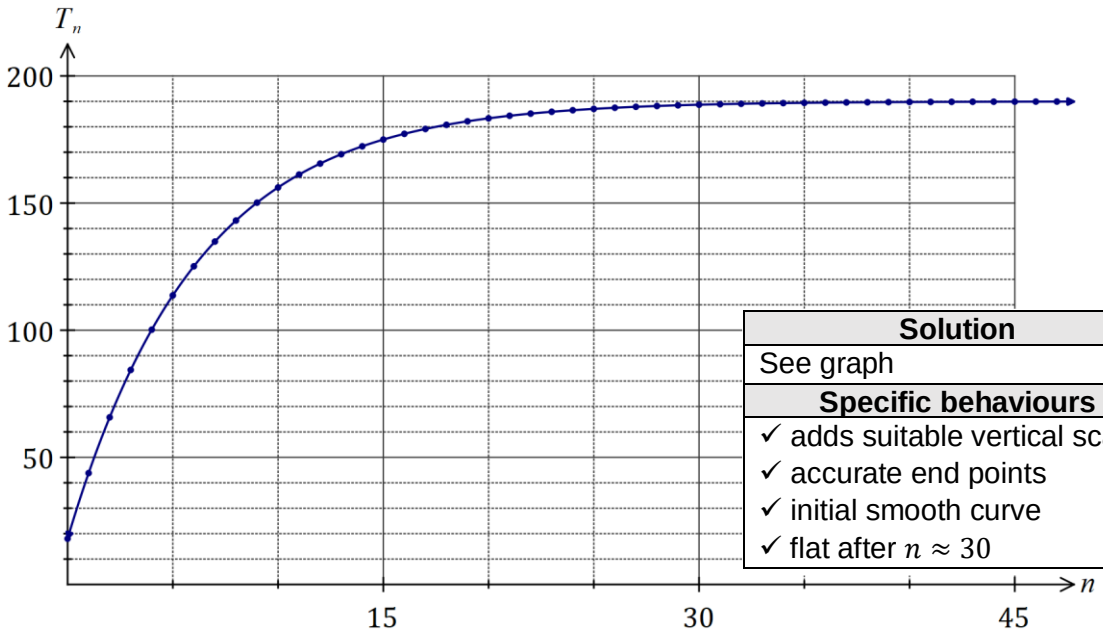
$$T_{n+1} = 0.85T_n + 28.5, \qquad T_0 = 18$$

(a) Use the recurrence relation to complete the table of values below, rounding the temperature to the nearest ℃. (2 marks)

n	0	5	10	15	20
T_n	18	114	156	175	183

Solution
See table
Specific behaviours
✓ at least 3 values correct, ✓ all values correct

(b) Sketch a graph of the temperature of the oven for the first 45 minutes on the axes below. Make sure you add a suitable scale to the vertical axis. (4 marks)



Solution
See graph
Specific behaviours
✓ adds suitable vertical scale
✓ accurate end points
✓ initial smooth curve
✓ flat after $n \approx 30$

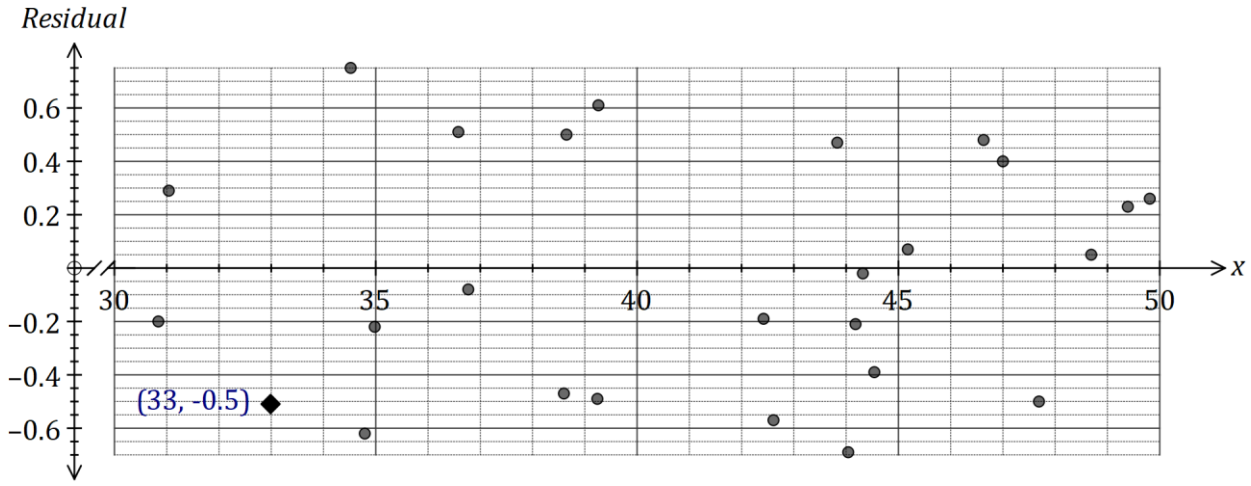
(c) The manufacturer claims that the oven will reach within 5 ℃ of its maximum temperature within 20 minutes of being turned on. Comment on this claim. (3 marks)

Solution
Maximum temperature is 190℃. Temperature exceeds 185℃ after 22 minutes. Claim is false.
Specific behaviours
✓ indicates maximum temperature
✓ states time reaches within 5℃
✓ comment on claim

See next page

Question 17 **(8 marks)**

A linear model was fitted to a set of data, resulting in a correlation coefficient of $r = -0.92$ and a least-squares line with equation $\hat{y} = 65.85 - 1.25x$. A residual plot for the linear model is shown below.



(a) Calculate, and add to the plot above, the residual for the point $x = 33$, $y = 24.1$. (3 marks)

Solution
$\hat{y} = 65.85 - 1.25(33) = 24.6$ $\text{Residual} = 24.1 - 24.6 = -0.5$
Specific behaviours
✓ calculates $\hat{y}$ ✓ calculates residual ✓ plots residual

(b) Use the residual plot to comment on the appropriateness of fitting a linear model to the data. (2 marks)

Solution
As no pattern is evident in the residual plot (as x values increase, residuals appear randomly above and below x-axis) then a linear model is appropriate.
Specific behaviours
✓ appropriate ✓ reason

(c) Determine the y-coordinate of the point with a residual of 0.4 on the above plot. (3 marks)

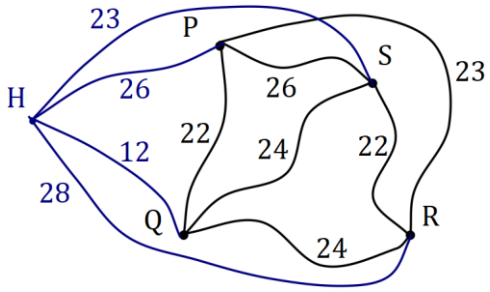
Solution
$x = 47$ $\hat{y} = 65.85 - 1.25(47) = 7.1$ $y - 7.1 = 0.4 \Rightarrow y = 7.5$
Specific behaviours
✓ indicates x-coordinate ✓ calculates $\hat{y}$ ✓ calculates y

See next page

Question 18 **(10 marks)**

Dara is visiting a city that has four museums: P, Q, R and S. The weights on the edges of the following graph represent the time, in minutes, that it takes to walk between the museums.

Solution (c)
See graph
Specific behaviours
✓ correct edges
✓ correct weights



(a) List, in the order visited, a set of vertices that form a Hamiltonian cycle in the graph. (2 marks)

Solution
$PQRSP$
Specific behaviours
✓ all vertices listed just once
✓ start and finish at same vertex

(b) Determine the shortest time it would take to leave P, walk to the other museums and return to P. (2 marks)

Solution
$PQSRP = 22 + 24 + 22 + 23 = 91$ m
Specific behaviours
✓ indicates vertices on a cycle from P
✓ shortest time

The time to walk from Dara's hotel, H, to museums P, Q, R and S is $26, 12, 28$ and 23 minutes respectively.

(c) Add vertex H and this information to the graph above. (2 marks)

(d) Dara plans to leave her hotel at 8.30 am, visit all the museums and then return to her hotel at 2.30 pm the same day. Determine the maximum total time she can spend inside the museums if she walks between them and describe the route she should take. (4 marks)

Solution
$HQPRSH = 12 + 22 + 23 + 22 + 23 = 102$ m 6 h $- 1$ h 42 m $= 4$ h 18 m $\quad (258$ m$)$
Specific behaviours
✓ lists possible route
✓ lists quickest route
✓ indicates quickest time
✓ calculates time in museums using h and m

End of questions

Supplementary page

Question number: _____

Supplementary page

Question number: _____

Supplementary page

Question number: _____